Structural Reliability: Rational Tools for Design Code Development

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Introduction

The basic problem of structural reliability is to ascertain that the strength *R* will be larger than the load (or load effects) *S* throughout the useful life of the structure. Due to uncertainties in the determination of strength and loads, reliability can only be established in probabilistic terms, i.e., $P(R > S)$. For real structures this is a complex issue, and as in every complex issue, guidance is needed in the design process which is provided through technical standards and codes.

Current design codes and standards (e.g. ASCE-SEI 7, ACI 318) are based on semi-probabilistic approaches. While it is agreed that the ideal would be to design a structure or structural element for a given probability of failure, the appeal of the semi-probabilistic approach stems from the coupling of design simplicity and the implicit incorporation of probabilistic concepts. While some practitioners understand the advantages in evolving towards semi-probabilistic code formats, a large number of structural engineers still see Structural Reliability as an unnecessary burden.

It is believed that for many engineers, much of their reluctance in accepting Structural Reliability and Reliability-Based Design comes from the lack of formal education in this subject of those professionals. As such, in this paper, Structural Reliability basics will be reviewed. Here, the major goal is to introduce this subject in a language that can be easily understood by the practitioner. Initially, the presence of uncertainties in almost all variables pertaining to the structural response (materials properties, geometries, predictive models, loads, etc.) will be discussed. In the sequence, methods for Reliability Analysis, -FOSM, FORM, SORM and also Monte Carlo simulation-, will be briefly presented. A number of attendant concepts (reliability index, probability of failure, performance function, design point, etc.) will be introduced. Levels of reliability methods and their relationships with current design codes will be examined. The importance of Structural Reliability concepts and methods in providing rational tools for design code development will be demonstrated from problems of current Structural Engineering practice.

Uncertainties in structural engineering

A number of uncertainties are present in the structural design problem. These may be related to inherent variability such as material properties (steel yield strength, steel ultimate strength, concrete compressive strength, concrete modulus of elasticity, etc.), dimensions (beam width and depth, concrete cover, etc.), loads (dead loads, live loads, wind, earthquake, etc.) or epistemic uncertainties, i.e., those related to the lack (or limited) knowledge. In this latter category are the errors associated to predictive models, sampling errors, and measurement errors. These errors may be reduced as more information is gained.

These uncertainties may be modeled as random variables. In this process, the associated mathematical models may be obtained from observational data. To this end, the histogram of the quantity of interest is plotted and a probability distribution is adjusted either by inspection or goodness of fit tests [1]. Fig. 1 shows the histogram and a superimposed Normal probability distribution (also known as Gauss distribution) for an 8-ksi (56 MPa) concrete. In a more general scenario, uncertainties are related to either spatial or temporal variability, or both.

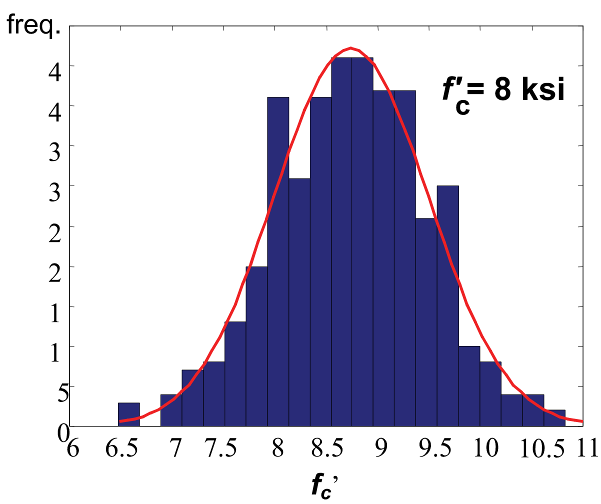

***Figure 1** – Histogram of concrete compressive strength and superimposed normal distribution.*

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Figure 2 shows the distribution of wind pressures over the envelope of a low-rise building for a given wind direction. In addition, each location is subjected to temporal variability, i.e., this quantity is described by a random process. Appropriate random variables associated to maximum or minimum values corresponding to the random process at hand may be obtained (see for instance, [2]).

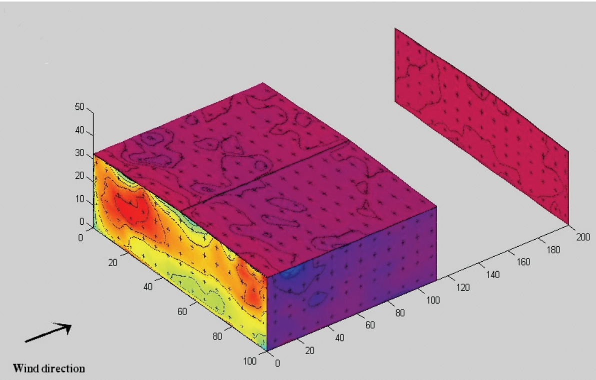


***Figure 2** – Spatial variability of wind pressures over the envelope of a low-rise building.*

Methods for reliability analysis

The basic problem of structural reliability is to ascertain that the strength *R* will be larger than the load (or load effects) *S* throughout the life of the structure, i.e., $P(R > S)$. Defining the safety margin as $M = R - S$, since *R* and *S* are random variables; *M* is also a random variable. Failure corresponds to the condition ($M < 0$) and the corresponding probability could be easily computed if the probability distribution associated to *M* is known. It can be shown [1] that for statistically independent *R* and *S* following Normal distributions, *M* is also normally distributed with mean μ_M and standard deviation σ_M :

$$\mu_M = \mu_R - \mu_S \quad (1)$$

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (2)$$

where μ_R and μ_S, μ_S and σ_S are the mean and standard deviation of variables *R* and *S*, respectively. In this case the probability of failure, P_f, can be computed in exact form as

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad (3)$$

where Φ is the cumulative distribution of the standard Normal variable and β , is the ratio:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (4)$$

From Eq. (4), it is seen that the ratio β is the number of standard deviations from the origin to the mean safety margin, μ_M . Since the larger the β the smaller the probability of failure, β is known as "reliability index" or "safety index". The probability density function (PDF) of the safety margin is presented in Fig. 3; also shown in this figure is the failure region ($M < 0$). A further observation of Eq. (4) is that in the space of reduced variables X_R' and X_M' (where $X' = (X - \mu)/\sigma$), β is the distance from the origin to the limit condition $M = 0$.

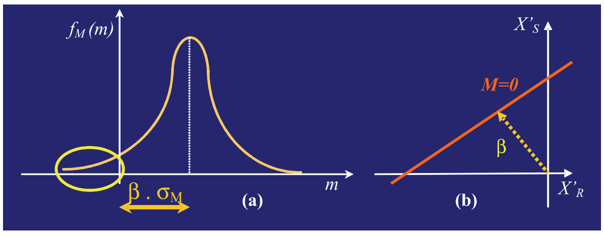


***Figure 3** – (a) pdf of the safety margin* M *and failure region (*M *< 0); (b) geometrical interpretation of the reliability index.*

In a more general problem, *R* and *S*, may be functions of multiple random variables, the basic variables $X_1, X_2, \ldots, X_n$. For each set of values of the basic variables it must be possible to state whether or not the structure has failed. In order to define the performance of a structure, a performance function $g(\boldsymbol{X}) = g(X_1, X_2, \ldots, X_n)$ is used. The limiting performance requirement may be defined as $g(\boldsymbol{X}) = 0$ and therefore $[g(\boldsymbol{X}) > 0]$ is the safe state and $[g(\boldsymbol{X}) < 0]$ is the failure state.

Figure 4 shows a limit state function in the space of reduced variables. For simplicity, only two variables, X_1' and X_2', are shown in the figure. As the limit-state surface (or failure surface) moves farther or closer to the origin, the safe region $g(\boldsymbol{X})>0$, increases or decreases accordingly. Therefore, a measure of the reliability of the system may be taken as the minimum distance from the origin of the reduced variables to the failure surface. The point $\boldsymbol{x}_i'^*$ ($x_1'^*, x_2'^*, \ldots, x_n'^*$) corresponding to this minimum distance is known as the "most probable failure point" (or "design point"). This minimum distance, which is the reliability index β, may be found through an optimization procedure which minimizes the distance D subjected to $g(\boldsymbol{X})=0$ (i.e., the design point belongs to the failure surface). Using the method of Lagrange multipliers it can be shown that, for uncorrelated variables, the reliability index β is given by

$$\beta = \frac{\sum_{i=1}^{n} x_i'^{*} \left(\frac{\partial g}{\partial X_i'}\right)_*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'}\right)_*^2}} \tag{5}$$

where $(\partial g/\partial X_i')_*$ are the partial derivatives evaluated at the design point. Since the design point is not known a priori, an iterative procedure may be used to compute the reliability index β. It can be shown that the computation of the reliability index through Eq. (5) is equivalent to the linearization of the performance function (i.e., first order expansion in a Taylor series) at the design point [3].

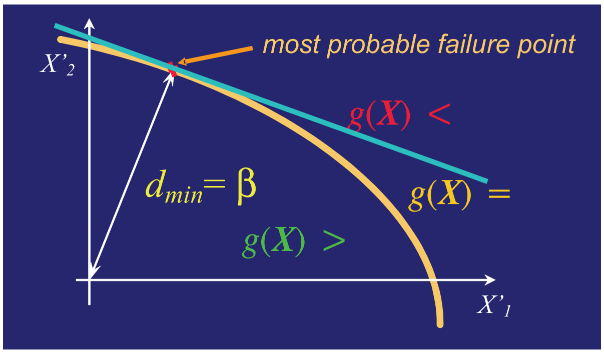


***Figure 4** – Failure surface, minimum distance, reliability index, most probable failure point and first order approximation.*

In the aforementioned procedure, the reliability index may be computed solely on information of the means and standard deviations of the basic variables. As such, this approach is known as "First Order Second Moment Method" (FOSM). If the probability distributions of all basic variables are known, then the probability of failure may be computed as $P_f \approx \Phi(-\beta)$. This procedure is consistent with uncorrelated Normal variables. For correlated and/or nonNormal variables a more involved process is required in the computation of β (see [3], [4]). In the literature this latter approach is known as "First Order Reliability Method" (FORM).

The linear approximation of nonlinear performance functions is equivalent to replacing an n-dimensional failure surface (a hyper-surface) with a hyper-plane tangent to the failure surface at the most probable failure point. As it can be seen in Fig. 4, this changes the boundary between the safe and the failure state. The reliability estimated on the basis of this approximation will be on the conservative or unconservative side depending on whether the actual failure surface is convex or concave toward the origin of reduced variables [3]. Improved estimates may be obtained by including second order terms in the Taylor series expansion of the performance function. This approach is known as "Second Order Reliability Method" (SORM).

It should be observed that for linear performance functions, i.e.,

$$g(\boldsymbol{X}) = a_0 + \sum_{i=1}^{n} a_i X_i , \tag{6}$$

the reliability index obtained from Eq. (5) is

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i \mu_{Xi}}{\sqrt{\sum_{i=1}^{n} (a_i \sigma_{Xi})^2}} . \tag{7}$$

In this case no approximation is required in the computation of the minimum distance. Furthermore, if the basic variables follow Normal distributions, $P_f = \Phi(-\beta)$, which means that the computed probability of failure is "exact" (in a mathematical sense). Also, it can be seen that Eq. (7) is a generalization of Eq. (4).

Monte Carlo Simulation

Monte Carlo Simulation involves repeating a simulation process, using in each simulation a particular set of values of the random variables generated in accordance with the corresponding probability distributions. By repeating the process, a sample of realizations, each corresponding to a different set of values of the random variables, is obtained. A sample from a Monte Carlo Simulation is similar to a sample of experimental observations [3]. Two items are required for a Monte Carlo Simulation: (1) a deterministic relation to describe the response of the structure, and (2) the probability distributions of all variables involved in calculation of the response. A key task in the Monte Carlo simulation is the generation of appropriate values of the random variables (i.e., random numbers). Procedures for the generation of random numbers are presented elsewhere [3].

The use of Monte Carlo simulation in the evaluation of structural performance may be twofold:

- computing the statistics (mean, standard deviation, and type of distribution) of the system response. In this case, first a sample of the structure response is obtained, then a probability distribution is fitted to the sample data and the distribution parameters are estimated;
- computing the probability of unsatisfactory performance of the structure. In this case, a performance function is established and a sample of the possible outcomes is simulated. The number of unsatisfactory performances is counted, and the probability of failure is obtained by the rate of unsatisfactory performances.

Example

Let's consider a beam, where the random variables are the yield stress Y, the plastic modulus Z, and the bending moment M. It is assumed that the probability distributions of Y, Z and M are Normal with parameters μ and σ, i.e., Y: N ($\mu_Y = 10$, $\sigma_Y = 0.8$); Z: N ($\mu_Z = 10$, $\sigma_Z = 0.4$); M: N ($\mu_M = 50$, $\sigma_M = 10$) in the corresponding units.

The performance function can be written as

$$g(\boldsymbol{X}) = Y Z - M \tag{8}$$

The computation of the probability of failure via Monte Carlo simulation is performed through the following steps:

1) Generation of random numbers for Y, Z and M. The histograms corresponding to samples of 100,000 realizations of Y, Z and M are shown in Fig 5. This can be easily performed using available commercial softwares;
2) A value for each random variable is extracted from the corresponding sample. These values are used in $g(\boldsymbol{X})$. This process is illustrated in Fig. 6;
3) The number of realizations in which the safe state $g(\boldsymbol{X}) > 0$ is violated is counted;

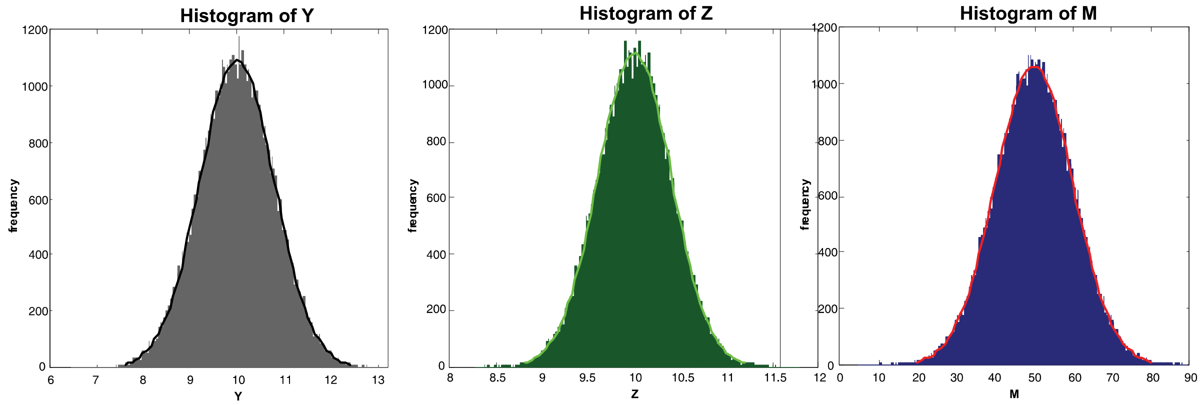
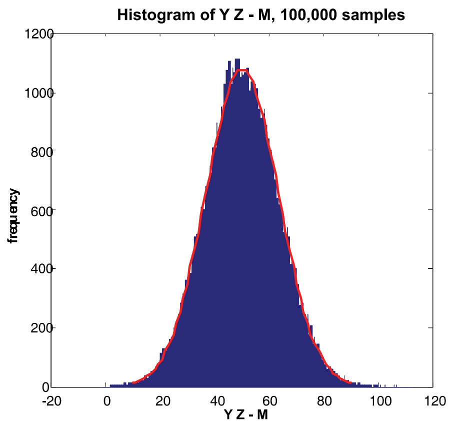


***Figure 5** – Histograms of* y, z, *and* m *and superimposed probability density functions.*



Element #	Y	Z	Y.Z	M	Saf. Margin
1	9.2146	9.729	89.6488	45.7152	43.9336
2	10.0045	10.2211	102.2576	50.4004	51.8572
3	9.7857	9.2571	90.5867	47.175	43.4116
	.	.	.	.	.
1516	7.4856	9.6671	72.3645	73.1292	-0.7647
	.	.	.	.	.
79500	7.6356	9.8046	74.8644	76.7178	-1.8534
	.	.	.	.	.
	.	.	.	.	.
100000	9.1851	9.9445	91.3411	26.3501	64.991

***Figure 6** – Histograms of* y z - m *and evaluation of the performance function.*

4) For example, if $g(\boldsymbol{X}) < 0$ is violated twice in 100,000 realizations, then $P_f = 2/100{,}000 = 2 \times 10^{-5}$.

As it can be seen, the so called "Crude Monte Carlo Simulation" is very simple. The accuracy of the results depends on the number of simulations performed, converging to the "exact" results as the sample size increases to infinite. In the above example, the computed probabilities of failure are 10^{-3}, 2×10^{-5}, 7.2×10^{-5} and 8.01×10^{-5}, for samples of 10^4, 10^5, 10^6, and 10^7 realizations, respectively. As such, a major drawback in this procedure is the need of large samples for the cases where small probabilities of failure are involved. In these cases more efficient procedures such as Importance Sampling may be used [4]. Also, efficiency is needed when the performance of the structure is established in terms of a numerical procedure (e.g., the Finite Element Method) rather than a single equation.

Levels of reliability methods

The great variety of idealizations in reliability models of structures, and the numerous ways in which it is possible to combine these idealizations to suit a particular problem, make it desirable to have a classification. Reliability methods are divided into levels, characterized by the extent of information about the structural problem that is used and provided [5]:

- Level 0: methods that use the allowable stress design format;
- Level 1: methods that employ only one "characteristic" value of each uncertain parameter (also known as semi-probabilistic methods). Load and resistance factor formats are examples of level 1 methods;
- Level 2: methods that employ two values of each uncertain parameter (commonly mean and variance), supplemented with a measure of the correlation between the parameters (usually covariance). These methods use the reliability index as a reference and are consistent with FOSM;
- Level 3: methods that employ probability of failure as a measure, and which therefore require the knowledge of the probability distribution of all uncertain parameters. These methods are consistent with FORM (or SORM) and Monte Carlo Simulation.
- Level 4: methods that explicitly account for risks (i.e., the product of probabilities of failure and consequences for all potential failure modes) in the assessment of life-cycle costs. The goal is the "Minimization of Life-Cycle Costs" or "Maximization of Net Benefits".

Applications

The last years have witnessed a steady evolution of design codes towards a better treatment of the uncertainties in the structural design problem. In this section, different possibilities for the utilization of Structural Reliability Methods in the development of design codes are presented. Nowadays it is imperative to consider all phases in the life of a structure (design, construction, inspection, maintenance, health monitoring, and rehabilitation), which requires implementation of probabilistic methods. As such, examples pertaining to some of these phases and the corresponding implications in the codification process are treated herein: code calibration, new materials, safety evaluation of existing structures, probabilistic design and life cycle methods.

Code Calibration

Current design codes and standards (e.g. ASCE-SEI 7, ACI 318, and Eurocodes) are based on semi-probabilistic approaches, i.e., level 1 methods. The rationale of a reliability method is a justification in terms of a higher level. Thus a level 1 method may be justified on level 2, in that it provides a reliability index that in some sense is close to a target value. A level 1 method may also be justified on level 3, in that it provides a probability of failure close to a target value. The parameters in the level 1 method (load and resistance factors) are then calibrated to resemble the selected higher level.

Code calibration usually involves a number of tasks (see for instance [6], [7], and [8]). A main problem in the calibration procedure is the selection of the target reliability index (or the target probability of failure). This has to be dealt within the context of risk acceptance criteria. A common approach is to base this decision on the values provided by current practice. Additionally, it should be emphasized that probabilistic methods incorporated in design codes are usually made via component reliability (beams, columns, slabs, etc. and single failure modes, e.g. shear, flexure, etc.) and not system reliability. The consequences of failure, which is a function of the importance of the component to structural integrity and the failure mode (ductile or fragile), may be dealt with by adopting different target values for different components (e.g. beams and columns) and different failure modes (e.g. flexure and shear).

New Materials

In the last decades, a number of new materials have caught the attention of the civil engineering community. While dealing with traditional materials, design guidelines have been based on several years of past experience. On the other hand, new materials, – and pertinent new concerns –, require new approaches in establishing the needed design guidelines. Structural Reliability methods provide powerful tools in the treatment of these problems.

In the case of high-strength concrete (HSC), the author has conducted an extensive research on the reliability of HSC columns ([9], [10], [11]). Regarding FRP, much still has to be done in the implementation of load and resistance factor procedures for FRP structures. For instance, the use of FRP bars as internal reinforcement in concrete structures is a promising alternative to steel bars. However, the mechanical behavior of FRP reinforcement differs from the behavior of steel reinforcement. While a ductile failure may be obtained in under-reinforced beams (steel bars), a brittle failure is unavoidable in FRP reinforced concrete (FRP-RC) beams. Therefore, the design of FRP-RC components demands a change in design philosophy. Due to the type of failure and increasing interest in FRP-RC structures, code provisions for the design of such structures, – in line with current reliability methods and tailored to the specificities of the materials involved –, shall be developed. The reliability assessment of FRP-RC beams is presented in [12]. In that work, the influence of concrete compressive strength, type of FRP bar, longitudinal reinforcement ratio, and load ratio in the resulting reliability levels are investigated.

Safety Evaluation of Existing Structures

Both developed and developing countries currently face the problem of managing an aging infrastructure. For instance, the 2005 ASCE Report Card for America's Infrastructure presents the figure of 27.1% of the 590,750 bridges in the USA as structurally deficient or functionally obsolete. As such, appropriate tools are required for the treatment of this problem.

The safety evaluation of existing structures is distinct from that related to the safety implementation in the design of new ones. While design codes for new structures allow for uncertainties in the design and construction processes, much of what was initially uncertain are no longer in the finished structure [4]. However, the determination of the actual values of various parameters (e .g., in situ concrete compressive strengths, concrete modulus of elasticity, etc.) in the existing structure introduces uncertainty of its own. Additionally, the structure may have undergone a deterioration process such as corrosion or fatigue; therefore, realistic structural performance can be established in probabilistic terms only. A question of utmost importance is the definition of criteria for the assessment of the service life of a given structure. Conservatism in the design of a new structure implies a small penalty in the costs; on the other hand, conservatism in the acceptance criteria for the existing structure may result in major impacts such as demolitions, extensive repairs, losses in the commercial activities, etc.

Probabilistic Design

For a given structural component and a given failure mode, code calibration of a semi-probabilistic method aims to achieve a uniform reliability. However, as pointed out in [13] the reliability analyses of the structures designed according to those codes reveal a fairly wide scatter. Even in the case of fully calibrated codes the limitations of the semi-probabilistic approach will always give rise to fluctuations in the corresponding probabilities of failure (or reliability indices). These problems can be circumvented by using probabilistic design. A probabilistic design code is currently under development by the Joint Committee on Structural Safety (JCSS). A draft of the "JCSS Probabilistic Model Code" is found at the site *www.jcss.ethz.ch*. It is expected that such a code will give guidance to researchers and engineers willing to perform a full probabilistic analysis and design of important structures.

Life Cycle Methods

The quest for a sustainable built environment has shifted the focus from costs incurred at erection time to all those incurred throughout the useful life of the structure. The life cycle cost analysis involves the estimation of initial, inspection, maintenance, and failure costs (among others). This in its turn encompasses the challenges related to systems reliability, time-dependent reliability (deterioration modeling and stochastic modeling of loads), structural health monitoring (inspection, maintenance, and repair), condition assessment of existing and retrofitted structures, and optimization. As aforementioned, the goal is this case may be "Minimization of Life-Cycle Costs" or "Maximization of Net Benefits". Since costs (and benefits) are incurred at different times they must be discounted to present time.

In these methods, the problem is treated in a rigorous and holistic manner. However, a number of problems must be dealt with: (i) the decision regarding the discount rate to be used in the analysis; (ii) the selection of the type of inspection to be considered, – constant or variable intervals –; (iii) the definition of the type of repair to be made; (iv) how to handle costs associated to the loss of human lives; (v) the estimation of probabilities of failure associated to all potential failure modes. In spite of the greater complexity embodied in these methods, they have been used in real problems such as the definition of strategies for bridge maintenance [14]. Additionally, it is expected that optimized designs may drastically reduce operational costs of the infrastructure [15].

Conclusions

The unprecedented development of computational capabilities, the increasingly available databases on materials and loads variability, the development of new sensor technologies, the use of new materials, the new level of maturity of probabilistic methods, and the many advances in the field of structural mechanics have paved the way for a more prominent role of Structural Reliability Methods as rational tools for design codes development. In this work, different alternatives for the incorporation of probabilistic methods in design codes have been reviewed. It is hoped that this may foster the understanding of this subject by the practitioners in the area of Structural Engineering.

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